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APPLICATION FOR LETTERS PATENT

**Distributed Topology Control for  
Wireless Multi-Hop Sensor Networks**

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**RELATED APPLICATIONS**

This patent application claims priority to U.S. provisional application serial no. 60/223,395 filed on August 07, 2000 and titled "Wireless Network Topology Control Methods and Arrangements", which is hereby incorporated by reference.

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**TECHNICAL FIELD**

The following description relates to network topology. More particularly, the following description pertains to providing distributed topology control to a multi-hop wireless network.

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**BACKGROUND**

A Multi-hop network is a collection of wireless mobile computing nodes dynamically forming a temporary network without the use of any existing network infrastructure or centralized administration. Multi-hop networks include packet-radio networks, ad-hoc networks, and sensor networks. As network size grows, communication between two nodes may go through multiple wireless links, or through multiple "hops" for one node to exchange data with another node across the wireless network. This multi-hop aspect occurs for a number of different reasons, for example, because of limited transmission range of wireless devices, communication path obstacles, spatial spectrum reuse, power saving considerations (the lifetime of a network that is operating on battery power is limited by the capacity of its energy source), and so on.

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Unlike wired networks that typically have fixed network topology (except in case of node failures), real-time physical displacement, or mobility of network

1 nodes in a multi-hop network is common because of the nature of the applications  
2 multi-hop networks are designed to support (e.g., applications for disaster  
3 recovery, battlefield, search and rescue, sensor nets, and so on). Because of node  
4 mobility, each node in a wireless network may change network topology by  
5 adjusting transmission power to control the set of one-hop neighbors, or nodes that  
6 are in communication with the node. Multi-hopping combined with a large  
7 network size, node mobility, device heterogeneity, bandwidth limitations, and  
8 battery power limitations make wireless multi-hop network topology control a  
9 major challenge.

10 The primary goal of topology control is to design power efficient  
11 algorithms that maintain network connectivity and optimize performance metrics  
12 such as network lifetime and throughput. Conventional techniques to provide  
13 wireless distributed multi-hop network topology control typically require  
14 positional information such as positional information that is acquired from a  
15 Global Positioning System (GPS) implementation at each node in the network.  
16 This positional information requirement for topology control is problematic for a  
17 number of reasons.

18 Even in an ideal environment acquisition of positional information may  
19 take a substantial amount of time—negatively impacting network response time  
20 and throughput. The actual amount of time that it takes to acquire positional  
21 information typically depends on how quickly each respective node can establish  
22 communication with multiple satellites and exchange data with the satellites to  
23 obtain corresponding positional data. Thus, respective nodes in the network may  
24 have to wait for an indeterminate, and potentially infinite amount of time before  
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1 receiving positional information. This is because positional information can only  
2 be reliably acquired in a limited number of environments.

3 Certain environments can completely block or substantially hamper satellite  
4 signals to nodes in a wireless multi-hop network. For instance, GPS signals are  
5 often undetectable or sporadic in indoor environments, in cloudy weather, in  
6 heavily treed areas, in cities, and so on. At best a node in such in environment will  
7 be able to obtain its required positional information. However, such environments  
8 often make communications between a node and the satellites irregular, which  
9 may cause a node to fade in and out of the network topology. At worst, a node  
10 may never receive its required positional information because it is blocked by  
11 environmental conditions that make satellite signal communication with network  
12 nodes impossible. In this worst case scenario, a node may become completely  
13 superfluous with respect to its role in the network.

14 Yet another problem, for example, with requiring positional information at  
15 each node in a multi-hop network to provide topology control is that technology  
16 required to acquire positional information is relatively expensive. Requiring such  
17 technology at each node in the network can substantially increase implementation  
18 costs of wireless multi-hop networks as well as corresponding maintenance costs.

19 Accordingly, the following described subject matter addresses these and  
20 other problems associated with providing distributed topology control for multi-  
21 hop wireless networks.

1 **SUMMARY**

2 The following description provides direction-based topology control to a  
3 distributed wireless multi-hop network. The network includes multiple potentially  
4 mobile nodes. Each node sends a discovery message in all directions. Each node  
5 discovers a set of neighboring nodes using a set of incoming signals from the  
6 neighboring nodes that are responsive to the discovery message. Responsive to  
7 receiving the incoming messages, each node makes a local decision about a  
8 substantially optimal transmission power with which to communicate with at least  
9 a portion of the discovered neighboring nodes. The decisions are based on the  
10 incoming signals and the decisions are made independent of any positional  
11 information (e.g., latitude and longitude). Each node in the network maintains  
12 communications with the decided portion of nodes to provide connectivity  
13 between each of the nodes.

14 In contrast to previous approaches that rely on knowing and sharing  
15 positional information of the nodes in the network to maintain connectivity in  
16 wireless multi-hop networks, the described subject matter provides a solution that  
17 relies on incoming directional information in the incoming signals from  
18 neighboring nodes. Advantageously, this novel technique increases network  
19 lifetime by allowing each node to locally determine an efficient power with which  
20 to communicate with specific ones of the other nodes in the network. At the same  
21 time this techniques provides for substantial if not complete connectivity with  
22 reasonable throughput in a wireless multi-hop network.

23 The described systems and procedures also include a number of optional  
24 enhancements to traditional topology control in wireless multi-hop networks,  
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including, for example, a techniques to reduce signal interference and enhance data throughput.

### **BRIEF DESCRIPTION OF THE DRAWINGS**

Fig. 1 shows an exemplary cone of coverage of degree  $\alpha$  that is centered on node  $u$ .

Fig. 2 shows an exemplary proof that a cone degree of  $\alpha \leq 5\pi/6$  provides a connected topology control in a wireless multi-hop network.

Fig. 3 shows a zoomed in aspect of Fig. 2 to provide an exemplary proof that a cone degree of  $\alpha \leq 5\pi/6$  substantially guarantees a connected multi-hop wireless network.

Fig. 4 shows a number of wireless multi-hop network nodes for which a cone-based topology algorithm using a cone degree of  $\alpha > 5\pi/6$  does not guarantee a connected multi-hop wireless network.

Fig. 5 illustrates a zoomed in portion of Fig. 4 to provide a “B-star” sub-graph that is shown to be disjoint from an A-star sub-graph of Fig. 4. Specifically, Fig. 5 further shows that using a cone degree of  $\alpha > 5\pi/6$  in the cone-based topology algorithm does not guarantee a connected multi-hop wireless network.

Fig. 6 illustrates a zoomed in portion of Fig. 4 to show that a cone coverage of  $\alpha = 5\pi/6 + \epsilon$  may result in a disjoint topology of the network nodes.

Fig. 7 shows that for cone degree of  $\alpha \leq 2\pi/3$ , all special edges in a wireless multi-hop network can be removed without resulting in a disjoint, or disconnected network topology—an edge  $(u, v)$  is called a special edge if node  $u$  is an  $i$ -neighbor of node  $v$ , but node  $v$  is an  $o$ -neighbor of  $u$ , or vice versa.

Fig. 8 is a diagram that is used to show that performing special edge removal for cone degrees where  $2\pi/3 < \alpha \leq 5\pi/6$  may disconnect nodes in the wireless multi-hop network.

Fig. 9 is an illustration for the pair-wise edge removal theorem, which is used to remove a redundant edge from a wireless multi-hop network topology.

Fig. 10 shows a topology graph of a wireless multi-hop network wherein no topology control is employed and every node transmits with the maximum power.

Fig. 11 shows a topology graph illustrating the effect of utilizing the cone-based algorithm with  $\alpha = 2\pi/3$  on the wireless multi-hop network of Fig. 10. Specifically, use of the cone-based algorithm with  $\alpha = 2\pi/3$  allows nodes in the dense areas of the topology to automatically reduce their transmission radius.

Fig. 12 illustrates a topology graph of Fig. 11 after the shrink-back operation has been performed allowing a number of boundary nodes to reduce their respective operational power.

Fig. 13 shows a topology graph of Fig. 12 after a shrink back operation and special edge removal has been applied to remove edges wherein a first node is within the radius of a second node, yet the second node is not within the radius of the first node.

Fig. 14 shows a topology graph of Fig. 13 after a shrink back, special edge removal, and pair-wise edge removal optimizations have been applied to remove redundant communication edges in the network.

Fig. 15 shows aspect of an exemplary wireless network node for providing distributed topology control to a wireless multi-hop network.

Fig. 16 shows an exemplary procedure for providing distributed topology control to a wireless multi-hop network.

## DETAILED DESCRIPTION

The following description sets forth exemplary subject matter providing distributed topology control to a multi-hop wireless sensor network. The subject matter is described with specificity in order to meet statutory requirements. However, the description itself is not intended to limit the scope of this patent. Rather, the inventors have contemplated that the claimed subject matter might also be embodied in other ways, to include different elements or combinations of elements similar to the ones described in this document, in conjunction with other present or future technologies.

### Overview

The following description puts forth a new approach to provide distributed topology control to a multi-hop wireless sensor network. Specifically, a novel distributed cone-based topology control algorithm is described that may considerably increase network lifetime and maintain global connectivity with reasonable throughput in a wireless multi-hop network. Network lifetime is affected by determining efficient transmitting radii for each node in the network to communicate with particular ones of the other network nodes. This is accomplished while substantially guaranteeing a same maximum connected node set as when all nodes are transmitting with full power. Additionally, in contrast to previous approaches for topology control in multi-hop networks that require node positional information, the described subject matter provides a solution that uses locally obtained directional information of other nodes to provide topology control to the network.



1 The distributed cone-based topology control algorithm includes a number  
2 of phases, which are summarized as follows. Starting with a small transmission  
3 radius, each node (denoted by node  $u$ ) broadcasts a neighbor-discovery message.  
4 Each receiving node acknowledges this broadcast message. Node  $u$  records all  
5 acknowledgments and the directions they came from. Node  $u$  then determines  
6 whether there is at least one neighbor in every cone of  $\alpha$  degrees, centered on  
7 Node  $u$ . In this first phase, Node  $u$  continues the neighbor discovering process by  
8 increasing its transmission radius (operational power) until either the above  
9 condition is met or an optimal termination power  $P$  (e.g., a power that is less than  
10 or equal to the nodes maximum transmission power) is reached. For  $\alpha$  smaller  
11 than or equal to  $5\pi/6$ , the algorithm substantially guarantees a maximum  
12 connected node set and power efficient connectivity between various node sets.

13 In an optional second phase, without impacting node connectivity, special  
14 and redundant edge removal processes are performed to reduce the node degrees  
15 and thereby reduce signal interference and data throughput.

### 16 **The Topology Control Problem**

17 The topology control problem is formalized as follows: A set  $V$  of possibly  
18 mobile nodes that are located in the Euclidean 2D plane. Each node  $u$  in the set of  
19  $V$  is specified by its coordinates,  $(x(u), y(u))$  at any given point in time. Each node  
20  $u$  has a power function  $p$  where  $p(d)$  gives the minimum power needed to establish  
21 a communication link to a node  $v$  at distance  $d$  away from  $u$ . Assume that the  
22 maximum transmission power  $P$  is the same for every node, and the maximum  
23 distance for any two nodes to communicate directly is  $R$ , i.e.  $p(R) = P$ . If every  
24 node transmits with power  $P$ , then we have an induced graph  $G_R = (V, E)$  where  
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1  $E = \{(u, v) | d(u, v) \leq R\}$ .  $G_R$  is connected. (When the actual network is not  
2 connected,  $G_R$  is the maximum connected sub-graph.) It is undesirable to have  $G_R$   
3 as the network topology for a couple of reasons. First, since the power required to  
4 transmit between nodes increases as the  $n$ th power of the distance between them,  
5 for some  $n \geq 2$ , it may require less power for a node  $u$  to relay messages through a  
6 series of intermediate nodes to  $v$  than to transmit directly to  $v$ . In addition, a node  
7 can potentially interfere with the transmission of any node  $w$  with  $p(d(u, w)) \leq P$ .

8       Topology control in a wireless multi-hop network is provided with respect  
9 to an embedded sub-graph  $G_e = (V, E_e)$  such that nodes transmit with smaller  
10 amounts of power as compared to traditional systems and techniques for providing  
11 topology control, while still substantially guaranteeing that  $G_e$  is connected.  
12 Additionally, each node has a number of neighbors (i.e., node degree) that is  
13 bounded by a small constant to reduce interferences and improve throughput.  
14 Moreover, each node  $u$  in the network can construct its neighbor set  $N(u) = \{v | (u,$   
15  $v) \text{ in the set of } G_e\}$  in a distributed fashion. Furthermore, if  $G_R$  changes to  $G'_R$  due  
16 to node failures or mobility, it is now possible to reconstruct a connected  $G'_e$   
17 without global coordination. In other words, it is now possible for each node to  
18 respond to failures and mobility issues by coordinating locally with "neighboring"  
19 nodes (i.e., other nodes within the node's various cones of coverage), rather than  
20 having to involve far away nodes.

21       To accomplish these aspects of distributed topology control for wireless  
22 multi-hop networks and to simplify deployment and reconfiguration upon failures  
23 and mobility, a novel cone-based topology control algorithm is presented that  
24 utilizes directional information (e.g., north, south, east, and west) in contrast to the  
25 positional information (e.g., latitude and longitude) that is typically required in

1 conventional topology control techniques. Additionally, this novel algorithm  
2 allows for asynchronous operations between the respective nodes in the distributed  
3 wireless multi-hop network. This means that the algorithm does not require all  
4 nodes to run the algorithm in a lock-step fashion.

### 5 **The Basic Cone-Based Topology Control (CBTC) Algorithm**

6 Three communication primitives: broadcast, send and receive, are defined  
7 as follows. The primitive  $\text{bcast}(p_0, m)_u$  is invoked by node  $u$  to send message  $m$   
8 with power  $p_0$  such that all nodes in set  $S = \{v | p(d(u, v)) \leq p_0\}$  will receive  $m$ ;  
9  $\text{send}(p_0, m)_{u,v}$  is invoked by node  $u$  to send message  $m$  to  $v$  with power  $p_0$ ;  
10  $\text{recv}(p_0', m)_{v,u}$  is used by  $v$  to receive  $m$  with reception power  $p_0'$ . Note that the  
11 reception power  $p_0'$  is smaller than the transmission power used by  $u$  to send  $m$   
12 due to radio signal attenuation in space. In addition, the  $\text{recv}(p_0', m)_{v,u}$  is null if  
13  $p_0 < p(d(u, v))$ . We assume that node  $v$  can derive  $p(d(u, v))$  given transmission  
14 power  $p_0$  and reception power  $p_0'$ . This assumption is reasonable in practice.

15 For ease of description, a synchronous model of communication between  
16 nodes in the network is first assumed. In a synchronous model, communication is  
17 governed by a global clock and proceeds in rounds, with each round taking one  
18 time unit. In each round each node  $u$  can examine the messages sent to it,  
19 compute, and send messages using *bcast* and *send* communication primitives. The  
20 communication channel is reliable. In later sections, this synchronous model  
21 assumption is relaxed, and the correctness of asynchronous executions of the  
22 algorithm is discussed.

TABLE 1 illustrates the Basic Cone-Based Topology Control (CBTC) algorithm with a cone degree of  $\alpha$ , hereinafter often called the basic CBTC- $\alpha$  algorithm, or the basic algorithm.

**TABLE 1**  
**The Basic CBTC- $\alpha$  Algorithm**

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 $N_u = \emptyset;$       //the neighbor set of  $u$ 
 $C = \emptyset;$       //the ordered cone set
 $p_u = \epsilon;$ 
 $A = \emptyset;$       //all the newly acquired neighbor IDs
 $\mathcal{V} = \emptyset;$     // the cones formed between  $u$  and all the
                    //newly acquired neighbor(s).

while ( $p_u < P$ ) {
     $p_u = \text{Increase}(p_u);$ 
     $\text{bcast}(p_u, m)_u$  and gather ACK responses;

     $N_u = N_u \cup A;$ 
     $C' = \text{makeCones}(A, \mathcal{V});$ 
    if ( $\text{computeUnionOfCoverage}(C, C') == 2\pi$ )
        break;
}

```

Fig. 1 shows an exemplary cone of coverage 100 of degree  $\alpha$  for a node  $u$ . The basic operation is for each node  $u$  to try to find at least one neighbor in every cone of degree  $\alpha$  centered at  $u$ .

To accomplish this, each node  $u$  starts running the CBTC- $\alpha$  algorithm by sending a node discovery message  $m$  in all directions. The message is sent with a small transmission power (e.g., represented by radius 102), which is gradually increased over time to discover more neighbors. Upon receiving the node discovery message  $m$  from node  $u$ , node  $v$  responds with an acknowledgement (ACK) message. Upon receiving the ACK from  $v$ , node  $u$  records  $v$  as a neighbor.

Each “acking” neighbor  $v$  will help cover “a cone” (e.g., cone 100 of Fig. 1), which centers on node  $u$  and spans degree  $\alpha/2$  in each direction. Node  $u$  continues to increase its power, thereby increasing its transmission radius 102, until either the union of all discovered node’s cone coverages span a circle of  $2\pi$ , or a substantially optimal terminating transmission radius has been reached.

The following definitions will be used throughout this description. The distance  $d(u, v)$  of two nodes  $u$  and  $v$  is their Euclidean distance. Since power is monotonic in distance, for simplicity, distance is often used rather than power. Significantly, notice that there is no requirement for knowing the global position (e.g., GPS information) of a node, nor is there any exact power function assumption given a certain distance. Node  $u$  considers node  $v$  as its neighbor when either  $v$  responds to  $u$ ’s request or vice versa. Node  $u$  maintains a bi-directional edge to particular ones or all of its neighbors, meaning that node  $u$  can both send messages to and receive messages from a neighboring node at the other end of the edge.

A circle that centers at  $u$  and has radius  $r$  is denoted by  $cir(u, r)$ . Node  $v$  is an in-radius neighbor (hereinafter, often referred to as an “ $i$ -neighbor”) of node  $u$  if  $d(u, v) \leq rad(u)$ . Node  $v$  is an out-of-radius neighbor (hereinafter, an out-of-radius neighbor is often referred to as an “ $o$ -neighbor”) of node  $u$  if  $d(u, v) > rad(u)$ . (Note that  $o$ -neighbors of  $u$  are those neighbors to which  $u$  has communicated acknowledgements to respective beacon requests). Given any two nodes  $u$  and  $v$ ,  $u$ ’s facing cone of degree  $\alpha$  towards  $v$  refers to the shaded area as shown in Fig. 1 and is denoted as cone  $(u, \alpha, v)$ .

If at the end of performing the cone-based algorithm A node  $b$  does not find neighbors to cover all cones of degree  $\alpha$ , then node  $b$  is a *boundary node*. (Note

that with the basic algorithm, the radius of any boundary node is initially the nodes maximum transmission radius.) Let  $(u, v)$  denote the edge between the two nodes  $u$  and  $v$ , and  $|(u, v)|$  denote the length of the edge. A path  $H$  is an ordered set of consecutive edges  $\{(u_0, u_1), (u_1, u_2), (u_2, u_3), \dots, (u_{k-1}, u_k)\}$ . A graph, or network topology is connected if there is a path from any node to any other node in the graph. Let  $G_r = (V, E_r)$  be the graph after every node finishes the execution of the basic cone-based algorithm.

### **Cone degree $\alpha \leq 5\pi/6$ Substantially Guarantees a Connected Graph**

For cone degree  $\alpha$  up to  $5\pi/6$  (i.e., 150 degrees), the local execution of the basic cone-based algorithm at each node collectively guarantees a connected graph. Specifically, any two nodes in  $G_R$  that are connected by an edge are connected by a path in  $G_r$ . Since  $G_R$  is connected,  $G_r$  must be connected. This demonstrates that cone-based topology control algorithm achieves a connected graph in a power-efficient way.

Lemma 2.1: For  $\alpha \leq 5\pi/6$ , given any two nodes  $A$  and  $B$  in  $G_r$ , if  $d(A, B) = d \leq R$  and there is no edge between  $A$  and  $B$ , there must exist  $A'$  and  $B'$  with  $d(A', B') < d(A, B)$  where  $A'$  ( $B'$ ) is either node  $A$  ( $B$ ) itself or one of its  $i$ -neighbors.

Proof: The fact that  $d(A, B) = d \leq R$ , but both  $rad(A)$  and  $rad(B)$  are less than  $d$  implies that  $A$  and  $B$  must have  $i$ -neighbor covering the facing cones  $cone(A, \alpha, B)$  and  $cone(B, \alpha, A)$ , respectively. The two cases are now distinguished.

- $\alpha \leq 2\pi/3$ : Node A must have an i-neighbor  $A_I$  inside cone  $(A, 2\pi/3, B)$ . Clearly,  $A_I$  must be inside  $\text{cir}(B, d(A, B))$  and so  $d(A_I, B) < d(A, B)$ . We choose  $A'$  to be  $A_I$  and  $B'$  to be B.
- $\alpha > 2\pi/3$ : If any of A's i-neighbors is/are inside cone  $(A, 2\pi/3, B)$  or vice versa, the same procedures are followed as discussed above to find  $A'$  and  $B'$ .

Fig. 2 shows an exemplary proof for asserting that a cone degree  $\alpha \leq 5\pi/6$  provides a connected topology graph of a wireless multi-hop network. Points p and q are the intersection points of  $\text{cir}(A, d)$  and  $\text{cir}(B, d)$ . Suppose there is no neighbor inside either cone  $(A, 2\pi/3, B)$  or cone  $(B, 2\pi/3, A)$ . To cover cone  $(B, \alpha, A)$ , node B must have an i-neighbor within either angle  $\angle hBq$  or  $\angle kBp$ , where h and k are chosen so that cone  $(B, \alpha, A)$  spans the  $\angle hBk$ . Without loss of generality, let Z be such a neighbor of B. Points p and q are the intersection points of  $\text{cir}(A, d)$  and  $\text{cir}(B, d)$ . Line  $rs$  and line  $tu$  both form  $\pi/2$  angles with line  $AB$ . If there are multiple such nodes, we choose Z to be the one that has the smallest  $\angle ZBA$ . Since  $\angle ZBu > 5\pi/6 \geq \alpha$ , there must be another i-neighbor Y of B within the angle  $\angle uBp$  to fully cover cone  $(B, \alpha, A)$ . If there are multiple such nodes, Y is selected to be the one with the smallest angle  $\angle YBA$ . Again, to fully cover cone  $(B, \alpha, A)$ ,  $\alpha > 2\pi/3$ , we must have  $\angle ZBY \leq \alpha$ . We apply the same arguments to node A to find its i-neighbors X and W with  $\angle XAW \leq \alpha$ . Note that at this point, either X or W can be the counterpart of Z.

In other words:

$$\angle WAB + \angle XAB \leq \alpha \leq 5\pi/6 \quad (1)$$

$$\angle ZBA + \angle YBA \leq \alpha \leq 5\pi/6 \quad (2)$$

1 If  $W$  is inside  $cir(B, d)$ , we simply choose  $W$  and  $B$  to be the new node pair.  
2 So we consider only  $d(W, B) \geq d$  for the rest of the proof. Similarly, we have  
3  $d(X, B) \geq d$ ,  $d(Y, A) \geq d$ , and  $d(Z, A) \geq d$ . We now prove, by contradiction, that  
4  $d(W, Z)$  and  $d(X, Y)$  cannot both be greater than or equal to  $d$ . The pair of nodes  
5 with distance less than  $d$  is chosen to be  $A'$  and  $B'$ . Suppose  $d(W, Z) \geq d$  and  $d(X,$   
6  $Y) \geq d$ . Let  $v$  be the intersection point of  $cir(Z, d)$  and  $cir(B, d)$ . Node  $W$  must be  
7 outside (or on) both circles  $(Z, d)$  and  $cir(B, d)$ , and so  $\angle WAB \geq \angle vAB$  as shown in  
8 Fig. 3.

9 Since  $d(v, Z) = d(v, B) = d(A, B) = d$ , and  $d(Z, B) < d$  and hence  $\angle ZBv >$   
10  $\pi/3$ , we have

$$11 \quad \angle vBA = \angle ZBA - \angle ZBv < \angle ZBA - \pi/3, \quad (3)$$

$$12 \quad \angle vBA = \pi - 2 * \angle vAB. \quad (4)$$

13 Accordingly,  $\angle ZBA - \pi/3 > \pi - 2 * \angle vAB$ ,  $\angle vAB > 2\pi/3 - \angle ZBA/2$ ,

$$14 \quad \angle WAB > 2\pi/3 - \angle ZBA/2. \quad (5)$$

15 By definition of  $Z$ ,  $\angle ZBA \leq \alpha/2 \leq 5\pi/12$  and so  $\angle WAB > 2\pi/3 - 5\pi/24 =$   
16  $11\pi/24 > \alpha/2$ . Therefore,  $W$  is not the counterpart of  $Z$  on the  $A$  side, and so  $X$   
17 must be  $Z$ 's counterpart. By replacing  $\angle ZBA$  with  $\angle XAB$  and  $\angle WAB$  with  $\angle YBA$ ,  
18 we have:

$$19 \quad \angle YBA > 2\pi/3 - \angle XAB/2. \quad (6)$$

20 From Equation 5 and 6, we have:

$$21 \quad \angle WAB + \angle XAB > (2\pi/3 - \angle ZBA/2) + (4\pi/3 - 2 * \angle YBA)$$

$$22 \quad = 2\pi - \angle ZBA/2 - 2 * \angle YBA. \quad (7)$$



Combine Equation (7) with Equation (1), and we have  $5\pi/6 > 2\pi - \angle ZBA/2 - 2 * \angle YBA$  and  $4 * \angle YBA + \angle ZBA > 7\pi/3$ . Combine that with Equation 2, we have  $3 * \angle YBA > 3\pi/2$  and  $\angle YBA > \pi/2$ , contradicting the fact that  $Y$  is inside  $\angle uBp$ .

Lemma 2.2: For  $\alpha \leq 5\pi/6$ , given any two nodes  $A$  and  $B$  in  $G_r$ , if  $d(A, B) = d \leq R$  and there is no edge between  $A$  and  $B$ , there must exist a path  $H$  between  $A$  and  $B$ .

Proof: Given nodes  $A$  and  $B$ , we apply Lemma 2.1 recursively to find node pairs with monotonically decreasing distances, and stop when a node pair is connected by an edge. This stopping condition is guaranteed to eventually happen because one does not find a node pair with distance less than  $L$ , the shortest distance among all node pairs in  $V$ . Connecting  $A$  and  $B$  constructs the path  $H$  through the nodes of the node pairs found in the process.

Theorem 2.3: Cone-based algorithm with degree  $\alpha$ ,  $\alpha \leq 5\pi/6$ , substantially guarantees a connected graph; that is, graph  $G_r$  is connected.

Proof: Given any edge  $(A, B)$  in  $G_r$ , we must have  $d(A, B) \leq R$ . If the edge does not exist in  $G_r$ ,  $A$  and  $B$  must be connected by a path according to Lemma 2.2. Since  $G_r$  is connected,  $G_r$  must be connected.

### **A Cone Degree of $5\pi/6$ is an Upper Bound**

Theorem 2.4: Cone-based algorithm with degree  $\alpha$ , where  $\alpha > 5\pi/6$ , does not guarantee a connected graph.

Proof: A counter example is constructed for any  $\alpha = 5\pi/6 + \epsilon$ ,  $\epsilon > 0$ . It suffices to prove the theorem only for an arbitrarily small  $\epsilon$  because the same counter example constructed also satisfies the angle constraints for any larger  $\epsilon$ .

Fig. 4 shows a disconnected topological graph of network nodes, wherein a cone degree of  $\alpha = 5\pi/6 + \epsilon$  is used to determine if there are neighboring nodes covering the cones. All circles of Fig. 4 have radius  $R$ , and only black points marked by capital letters are actual nodes. Specifically, Fig. 4 illustrates the basic shape of the counter example: the distance between node  $A$  and node  $B$  is set to be  $R$ . So  $G_R$  is connected.

In  $G_r$ , the sub-graph on the left-hand side consists of nodes  $A$ ,  $W$ ,  $X$ , and  $K$  and edges  $AW$ ,  $AX$ , and  $AK$  (hereinafter, referred to as  $A$ -star sub-graph), will be shown to be disjoint from a similar  $B$ -star sub-graph that is illustrated in Fig. 4. Circles  $(B,R)$  and  $(A,R)$  intersect at points  $q$  and  $p$ . Lines  $rs$  and  $tu$  are both orthogonal to line  $AB$ . Nodes  $Z$  and  $X$  are chosen based on the given  $\epsilon$ , which is to be described shortly. Node  $W'$  (See Fig. 6) is at the intersection of circle  $(Z,R)$  and line  $rs$ . Similarly, node  $Y'$  (not shown) is at the intersection of  $(X,R)$  and line  $tu$ . Finally, we choose nodes  $K$  and  $J$  so that  $\angle WAK = \angle YBJ = 7\pi/12$  and length  $AK = BJ = R/2$ .

Given an arbitrarily small angle of degree  $\epsilon$  ( $\epsilon < \pi/6$ ), we choose node  $Z$  as follows. Referring to Fig. 5 (a zoomed in portion of Fig. 4), we choose point  $q'$  on the circle  $(B,R)$  so that the angle  $\angle qBq'$  is of degree  $\epsilon$ . Draw a line  $qo$  that is parallel to  $AB$ . Since  $\angle oqB = \angle qBA = \pi/3$ , we have  $\angle q'qo = (\pi - \epsilon)/2 - \pi/3 = \pi/6 - \epsilon/2 > 0$ . We choose  $Z$  on line  $qo$  so that  $\angle q'Zq = \pi/2$ . Clearly, we have length  $ZB < R$  and  $ZA > qA = R$ . So node  $W'$  (with  $ZW' = R$  by definition) must be located above node  $A$ . We choose node  $W$  to be halfway between  $A$  and  $W'$  so that

1  $WZ > R$ . Node  $X$  is similarly chosen so that  $XA < R$ ,  $XB > R$ , and node  $Y$  is  
2 located halfway between  $B$  and  $Y'$ .

3 Now we consider the execution of the cone-based algorithm with  $\alpha = 5\pi/6 +$   
4  $\epsilon$ ,  $\epsilon > 0$ , at every node in Fig. 4 by referring also to Figs. 5 and 6. Node  $B$  stops  
5 the execution at radius  $ZB < R$  because  $\angle ZBY < \angle q'Bu = 5\pi/6 + \epsilon$  and  $\angle ZBJ <$   
6  $\angle qBJ = 7\pi/12 = \angle JBY < 5\pi/6 + \epsilon$ . Node  $W$  stops only when reaching radius  $R$   
7 because it is more than distance  $R$  away from any node of  $B$ -star. Nodes  $X$  and  $K$   
8 also stop at radius  $R$  for the same reason. Same arguments apply to nodes of  
9  $B$ -star. Therefore,  $A$ -star and  $B$ -star are disjoint after each node finishes running  
10 the cone-based algorithm with a cone degree of  $\alpha = 5\pi/6 + \epsilon$ .

11 Accordingly, the cone-based algorithm with degree  $\alpha$ ,  $\alpha > 5\pi/6$ , may result  
12 in a disconnected graph  $Gr$  as shown in Fig. 4, and hence does not guarantee a  
13 connected graph.

#### 14 **Optimizations to the Basic Algorithm**

15 In this section, three optimizations to the basic algorithm are described: a  
16 shrink-back operation, special edge removal, and pair-wise edge removal. These  
17 optimizations do not disconnect  $Gr$ , but rather preserve a connected graph while  
18 potentially reducing a node's transmission power and interference (thereby  
19 potentially increasing data throughput in the network).

##### 20 **The shrink-back operation**

21 In the basic algorithm, all boundary nodes stay with the maximum radius  $R$   
22 after failing to find  $i$ -neighbors to cover all cones. A shrinking phase at the end of  
23 the growing phase is to allow each boundary node to shrink back its radius until  
24 any further shrinking would reduce the overall cone coverage.  
25

Such shrink-back operation allows a boundary node to exclude those i-neighbors that it has unnecessarily acquired as part of a failed attempt to find i-neighbors in other directions. After a boundary node locally determines the new radius that it should shrink back to, it sends a neighbor revocation beacon to those i-neighbors that are no longer inside its newly determined radius and decreases its transmitting power accordingly. We prove in the following theorem that the shrink-back operation does not disconnect  $G_r$ .

Definition 3.1: Let  $G_s = (V, E_s)$  be the graph after every node finishes the execution of the basic cone-based algorithm and the shrink-back operation.

Theorem 3.2: For  $\alpha \leq 5\pi/6$ , the basic cone-based algorithm with the shrink-back optimization still substantially guarantees a connected graph.

Proof: Given any two nodes  $A$  and  $B$  in  $G_s$ , if  $d(A, B) = d \leq R$  and there is no edge between  $A$  and  $B$ , either both  $A$  and  $B$  never reached radius  $d$  in the growing phase or one or both of them reached  $d$  and then shrunk back past  $d$ . In either case, nodes  $A$  and  $B$  must still have i-neighbors fully covering the cones  $\text{cone}(A, \alpha, B)$  and  $\text{cone}(B, \alpha, A)$ , respectively, because any shrink-back operation can only remove those i-neighbors that provide redundant cone coverage. Therefore, the proofs of Lemma 2.1, Lemma 2.2, and Theorem 2.3 are all still valid, and  $G_s$  is connected.

### Special edge removal

Maintaining o-neighbors is undesirable in practice because such neighbors may require special treatments in broadcast operations. For example, suppose node  $u$  has a very small radius  $\text{rad}(u)$ , but has one o-neighbor  $v$  with  $d(u, v)$  close to  $R$ . To send a broadcast message  $m$ , node  $u$  can send the message using

1  $bcast(p(rad(u)), m)_u$  to reach all i-neighbors, but must send a separate message  
2 with a much larger power to reach  $v$  using  $send(p(d(u, v)), m)_{u,v}$ .

3 Definition 3.3: An edge  $(u, v)$  is called a special edge if  $u$  is an i-neighbor  
4 of  $v$  but  $v$  is an o-neighbor of  $u$ , or vice versa. We denote the edge as  $se(v, u)$  for  
5 the former case, and  $se(u, v)$  for the latter.

6 In this section, we prove that, for  $\alpha \leq 2\pi/3$ , all special edges can be removed  
7 without disconnecting the graph. We also show that special edges can not be  
8 removed for any  $\alpha$ ,  $2\pi/3 < \alpha \leq 5\pi/6$ , by demonstrating the procedure for  
9 constructing counter examples in which removing a special edge would disconnect  
10 the graph. The following lemma is an extension of Case 1 in the proof of  
11 Lemma 2.1.

12 Lemma 3.4: For  $\alpha \leq 2\pi/3$ , given any two nodes  $u$  and  $v$  with  $d(u, v) \leq R$ , if  
13  $rad(u) < d(u, v)$ , there must exist a path  $H$  between  $u$  and  $v$  consisting of edges  $(u_0,$   
14  $u_1), (u_1, u_2), (u_2, u_3), \dots, (u_{k-1}, u_k)$  ( $u_0 = u$  and  $u_k = v$ ), which have the following  
15 properties for all  $i = 1, \dots, k$ :

- 16 •  $u_i$  is an i-neighbor of  $u_{i-1}$ ;
- 17 •  $d(u_{i-1}, u_i) < d(u, v)$ ; that is, all edges on  $H$  are shorter than  $d(u, v)$ ;
- 18 and,
- 19 •  $d(u_i, v) < d(u_{i-1}, v)$ ; that is,  $u_i$  is closer to  $v$  than  $u_{i-1}$ .

20 Fig. 7 shows that for cone degree of  $\alpha \leq 2\pi/3$ , all special edges in a wireless  
21 multi-hop network can be removed without disconnecting the topological graph—  
22 an edge  $(u, v)$  is called a special edge if  $u$  is an i-neighbor of  $v$  but  $v$  is an  
23 o-neighbor of  $u$ , or vice versa. (All dotted circles in Fig. 7 have radius  $d$ ).  
24 Because  $rad(u) < d(u, v)$  implies that there exists an i-neighbor  $u_i$  of  $u$  within the  
25

1  $2\pi/3 \angle qup$ . Therefore,  $cone(u, \alpha, v)$  can be covered. Clearly,  $d(u, u_1) \leq rad(u) <$   
2  $d(u, v)$  and  $d(u_1, v) < d(u, v)$ .

3 Applying the same argument iteratively on the intermediate node pair  $u_{i-1}$   
4 and  $v$ , we have (1)  $u_i$  is an  $i$ -neighbor of  $u_{i-1}$  and (2)  $d(u_{i-1}, u_i) \leq rad(u_{i-1}) < d(u_{i-1}, v)$   
5  $< d(u_{i-2}, v) < \dots < d(u, v)$ . Since  $d(u_i, v)$  is monotonically decreasing and there is  
6 a lower bound  $L$  on the inter-node distance, we must eventually reach a  $u_{k-1}$  that  
7 includes  $v$  as an  $i$ -neighbor to complete the path  $H$ .

8 Theorem 3.5: For  $\alpha \leq 2\pi/3$ , all special edges can be removed without  
9 disconnecting the graph.

10 Proof: A path consisting of only non-special edges must also connect every  
11 pair of nodes that are connected by a special edge. So all special edges can be  
12 removed without disconnecting the graph. All of the special edges are sorted  
13 based on their lengths in non-decreasing order and denoted as  $e_1, e_2, \dots, e_m$ , where  
14  $|e_i| \leq |e_{i+1}|$  and  $m$  is the total number of special edges.

15 By induction, every special edge  $e_k = se(v_k, u_k)$  has a corresponding path  
16  $H'_k$ , which connects  $v_k$  and  $u_k$  and consists of only non-special edges. Examining  
17  $e_1 = se(v_1, u_1)$  where  $rad(u_1) < d(u_1, v_1)$ , from Lemma 3.4, there must exist a path  
18  $H_1$  between  $u_1$  and  $v_1$ , which consists of only edges that are shorter than  $|e_1|$ . Since  
19  $e_1$  is the shortest among all special edges, all edges on  $H_1$  must be non-special  
20 edges. Let  $H'_1 = H_1$  and we have the induction step  $k = 1$ .

21 Suppose, for every  $e_j = se(v_j, u_j)$ ,  $1 \leq j \leq i-1$ , we have found a path  $H'_j$   
22 between  $u_j$  and  $v_j$ , which consists of only non-special edges. Now we consider  $e_i$   
23  $= se(v_i, u_i)$ , the induction step  $k = i$ . From Lemma 3.4, there exists a path  $H_i$   
24 between  $u_i$  and  $v_i$ . If  $H_i$  contains any special edge  $e_j$ , we must have  $j \leq i - 1$ .  
25

Replacing every such  $e_j$  with its corresponding  $H'_j$  yields a path  $H'_i$  that connects  $u_i$  and  $v_i$  through only non-special edges.

Now all special edges can be removed without disconnecting any of the  $H'_k$ 's edges. So the graph remains connected.

Theorem 3.6: For  $2\pi/3 < \alpha \leq 5\pi/6$ , performing special edge removal may disconnect the graph.

Proof: Fig. 8 is a diagram that is used to show that for cone degrees where  $2\pi/3 < \alpha \leq 5\pi/6$ , performing special edge removal may disconnect the graph. Given any  $\alpha = 2\pi/3 + \varepsilon$ ,  $2\pi/3 < \alpha \leq 5\pi/6$ , we demonstrate the procedure for constructing the counter example shown in Fig. 8, in which  $se(B,A)$  is a special edge and removing  $se(B,A)$  would disconnect the graph. Let  $d(A,B) = R$ . Position  $W$  so that  $\angle WAB = \pi/3 + \varepsilon/2$  and  $\angle BWA = \angle BQA = \pi/3$ . Since  $\angle WBA = \pi/3 - \varepsilon/2 < \angle BWA < \angle WAB$ , we have  $d(A,W) < d(A,B) < d(B,W)$ . Position  $X$  in a similar way, and we have  $\angle WAX = \alpha$ . Finally, let  $d(A, V) \leq d(A,W)$ .

The cone-based algorithm is executed with shrink-back optimization on every node. Node  $A$  stops at  $rad(A) = d(A,W)$ . Nodes  $W$ ,  $X$ , and  $V$  are boundary nodes, and each of them grows its radius to  $R$  and then shrinks back. Since  $d(B,X) = d(B,W) > d(A,B) = R$  and  $d(B, V) > R$ , none of them can reach node  $B$ . Node  $B$  grows its radius to  $R$ , obtains  $A$  as its  $i$ -neighbor, and stays at  $rad(B) = R$ . Clearly,  $se(B,A)$  is a special edge and the graph in Fig. 8 is connected. Removing  $se(B,A)$  would, however, disconnect node  $B$  from the rest of the graph.

The basic cone-based algorithm with shrink-back optimization is enhanced so that each node can locally detect and remove special edges for  $\alpha \leq 2\pi/3$ . For any  $se(B,A)$ , node  $A$  can detect that  $se(B,A)$  is a special edge simply by observing

that  $rad(A) < d(A,B)$ . However, node  $B$  may not be able to determine that because it does not know whether node  $A$  has finished its execution of the algorithm or not. To supply that information, the algorithm is enhanced as follows. Upon acknowledging a beacon request from  $B$ , node  $A$  includes  $B$  in its o-neighbor set unless  $B$  is already an i-neighbor of  $A$ . After  $A$  finishes the basic algorithm and the shrink-back operation,  $A$  sends  $rad(A)$  to all the nodes in the o-neighbor set.

### Pair-wise edge removal

Another optimization aims at further reducing the number of neighbors for each node and places an upper bound on the node degree. Each node is assigned a unique ID, which is included in messages from the node. Given any pair of node  $u$  neighbors such as node  $v$  and node  $w$ , node  $u$  determines which of them is closer as follows.

Recall that node  $u$  grows its radius in discrete steps. It includes its transmission power level in each beacon request so that the receivers can use the same power to respond to reach node  $u$ . A node that responds to  $u$ 's request in an earlier step is clearly closer to  $u$  than those that respond in later steps. If  $v$  and  $w$  both respond to  $u$ 's request with power level  $p'(u)$  and  $w$ 's response has a lower power level when it is received by  $u$ , then we must have  $d(u,w) > d(u,v)$ .

We now define edge IDs and the notion of redundant edge.

**Definition 3.7:** Each edge  $(u, v)$  is assigned an edge ID  $eid(u, v) = (i_1, i_2, i_3)$  where  $i_1 = d(u, v)$ ,  $i_2 = \max(\text{node IDs of } u \text{ and } v)$ , and  $i_3 = \min(\text{node IDs of } u \text{ and } v)$ . Comparison of two edge IDs is based on the lexicographical order.

**Definition 3.8:** Given any  $\theta \leq \pi/3$  and given any pair of edges  $(u, v)$  and  $(u,w)$  such that  $\angle vuw < \theta$ , if  $eid(u, v) > eid(u,w)$ , then  $(u, v)$  is called a redundant



edge. (Note that the  $\pi/3$  upper bound on  $\theta$  is to make sure that, if edge  $(v,w)$  exists, it is not longer than both  $(u, v)$  and  $(u,w)$ .)

Depending on the metrics to optimize, a redundant edge may or may not be removed. We call the optimization that removes all redundant edges the pair-wise edge removal optimization.

Theorem 3.9: For  $\alpha \leq 2\pi/3$ , all redundant edges can be removed without disconnecting the graph. Proof. Every pair of nodes that are connected by a redundant edge must also be connected by a path consisting of only edges that are not redundant edges in any pair of edges of a node. So all redundant edges can be removed without disconnecting the graph. Given all the edges, each is a redundant edge in at least one pair of edges, we sort them based on their edge IDs in non-decreasing order and denote them  $e_1, e_2, \dots, e_m$ , where  $|e_i| \leq |e_{i+1}|$  and  $m$  is the total number of redundant edges.

By induction, every redundant edge  $e_k = (u_k, v_k)$  has a corresponding path  $H'_k$ , which connects  $u_k$  and  $v_k$  and contains no redundant edges. Examining  $e_1 = (u_1, v_1)$ , by definition there must exist an edge  $(u_1, w_1)$  such that  $\angle v_1 u_1 w_1 < \theta \leq \pi/3$  and  $eid(u_1, v_1) > eid(u_1, w_1)$ , as shown in Fig. 9, which is an illustration for the pair-wise edge removal theorem, and wherein  $\angle v_1 u_1 w_1 < \pi/3$ , wherein edge  $(u_1, v_1)$  is a redundant edge.

Since  $e_1$  is the redundant edge with the smallest edge ID,  $(u_1, w_1)$  is not a redundant edge. The fact  $\angle v_1 u_1 w_1 < \pi/3$  implies that the distance between  $v_1$  and  $w_1$  must be shorter than  $d(u_1, v_1)$ . If edge  $(w_1, v_1)$  exists, the edge is not be a redundant edge, and so  $H'_1$  includes  $(u_1, w_1)$  and  $(w_1, v_1)$ . If there is no edge between  $v_1$  and  $w_1$ , since  $d(v_1, w_1) < d(u_1, v_1) \leq R$  and  $\alpha \leq 2\pi/3$ , there must exist a

1 path  $H_l$  between  $v_l$  and  $w_l$ , which consists of only edges shorter than  $d(v_l, w_l)$ ,  
2 according to Lemma 3.4. Clearly, no edges on  $H_l$  can be a redundant edge and so  
3 we can connect  $(u_l, w_l)$  with  $H_l$  to obtain  $H'_l$ , thus finishing the induction step  $k =$   
4  $l$ .

5 Suppose, for every  $e_j = (u_j, v_j)$ ,  $1 \leq j \leq i - 1$ , we have found a path  $H'_j$   
6 between  $u_j$  and  $v_j$ , which contains no redundant edge. Now we consider  $e_i =$   
7  $(u_i, v_i)$ , the induction step  $k = i$ . By definition, there exists another edge  $(u_i, w_i)$   
8 with  $eid(u_i, v_i) > eid(u_i, w_i)$  and  $\angle v_i u_i w_i < \pi/3$ . If  $(u_i, w_i)$  is a redundant edge, it  
9 must be one of  $e_j$ 's, where  $j \leq i - 1$ . If the path  $H_i$  (from Lemma 3.4) between  $v_i$   
10 and  $w_i$  contains any redundant edge  $e_j$ , we must have  $|e_j| < |e_i|$  and so  $j \leq i - 1$ . By  
11 connecting  $(u_i, w_i)$  with  $H_i$  and replacing every redundant edge  $e_j$  on the path with  
12  $H'_j$ , a path  $H'_i$  between  $u_i$  and  $v_i$  is obtained, which contains no redundant edges.

13 Thus all redundant edges are removed without disconnecting any of the  
14  $H'_k$ 's, and the graph remains connected.

15 Theorem 3.10: Pair-wise edge removal with  $\theta \leq \pi/3$  places an upper bound  
16 of  $2\pi/\theta$  on the node degree.

17 Proof by contradiction: Suppose there is a node that has  $n$  neighbors, where  
18  $n > 2\pi/\theta$  after pair-wise edge removal. Let  $\beta$  be the smallest angle between any  
19 pairs of edges. We must have  $\beta \leq 2\pi/n < \theta$ . By definition, one of the edges for  
20 the  $\beta$  angle must be a redundant edge and should have been removed. Thus, there  
21 is a contradiction. Therefore, pair-wise edge removal with  $\theta \leq \pi/3$  places an upper  
22 bound of  $2\pi/\theta$  on the node degree.

## Network Reconfiguration, Node Failures, and Asynchrony

In a multi-hop wireless network, nodes can be mobile. Even if a node does not move, a node may cease operations if it runs out of energy stores. Additionally, a new node may be added to the network. Such events may change overall the network's topology, or configuration. To manage such changes in a wireless multi-hop network, each node periodically communicates a "still alive" message, or beaconing message to neighboring nodes to indicate that the communicating node is operational. Any one of a number of different protocols such as the Neighbor Discovery Protocol (NDP) can be used for such communications.

Each beacon message includes a set of information corresponding to the communicating node such as the communicating node's ID and the transmission power of the beacon. A neighbor node is considered failed if a pre-defined number of beacons from the node are not received within a particular amount of time  $\tau$ . A node  $v$  is considered a new neighbor of  $u$  if a beacon is received from  $v$  and no beacon was received from  $v$  during a previous time  $\tau$  interval (this interval may or may not be the same as the time  $\tau$  that it takes to determined that a node has failed).

A node  $u$  broadcasts, or beacons with sufficient power to reach all of its neighbors in  $N_u$  for reconfiguration based on the basic cone-based algorithm. Specifically, if node  $u$  beacons with power  $p_u^b$  where  $p_u^b$  is the power that  $u$  must use to reach all its neighbors in  $N_u$ , (including  $i$ -neighbors and  $o$ -neighbors)— $p_u^b \geq p(\text{rad}(u))$ , which is the power to reach all  $i$ -neighbors, then this is sufficient power to beacon using the basic cone-based algorithm. (Such beaconing can be

combined with asymmetric edge removal if  $\alpha \leq 2\pi/3$ , in which case power  $p_u$  is used).

Three basic network reconfiguration indicating events are defined as follows:

- A  $join_u(v)$  event happens when node  $u$  detects a beacon from node  $v$  for the first time;
- A  $leave_u(v)$  event happens when node  $u$  misses some predetermined number of beacons from node  $v$ ;
- An  $angleChange_u(v)$  event happens when  $u$  detects that  $v$ 's angle with respect to  $u$  has changed. (Note this could be due to movement by either  $u$  or  $v$ .)

The reconfiguration algorithm: It is assumed that each node is tagged with the power used when it was first discovered, as discussed above with respect to the shrink-back operation. (This assumption is not necessary, but it minimizes the number of times that CBTC needs to be rerun). If a  $leave_u(v)$  event happens, and if the union of the cone coverage reduces after dropping the  $cone(u, \alpha, v)$  from the set of cones  $C$ , node  $u$  reruns CBTC- $\alpha$ , starting with power  $p_u^b$ .

If a  $join_u(v)$  event happens,  $u$  computes its  $cone(u, \alpha, v)$  and the power needed to reach  $v$ . As in the shrink-back operation,  $u$  then removes nodes, starting with the farthest neighbor nodes and working back, as long as their removal does not change the coverage.

If an  $angleChange_u(v)$  event happens, node  $u$  modifies the  $cone(u, \alpha, v)$  in the cone set  $C$ . The cone coverage reduces, then CBTC- $\alpha$  is rerun, again starting

1 with power  $p_u^b$ . Otherwise, nodes are removed, as in the shrink-back operation, to  
2 see if less power can be used.

3       There may be more than one change event that is detected at a given time  
4 by a node  $u$ . For example, if  $u$  moves, then there will be in general several *leave*,  
5 *join* and *angleChange* events detected by  $u$ . If more than one change event is  
6 detected by  $u$ , changes discussed above are performed as if the events are observed  
7 in some order, as long as there is no need to rerun CBTC. If CBTC needs to be  
8 rerun, node  $u$  deals with all changes simultaneously.

9       The reconfiguration algorithm substantially guarantees that each cone of  
10 degree  $\alpha$  around a node  $u$  is covered (except for boundary nodes), just as the basic  
11 algorithm does. However, frequent network topology changes may cause the  
12 actual connectivity in the network to be in a state of flux. Yet, if changes within  
13 the network stabilize, or cease for some amount of time, then the reconfiguration  
14 algorithm may catch up with the changes and subsequently maintain connectivity  
15 of the stabilized network. The proof that the reconfiguration algorithm maintains  
16 connectivity follows immediately from the proof of Theorem 2.3.

17       This reconfiguration algorithm works in combination with the basic  
18 algorithm CBTC- $\alpha$  and in combination with the asymmetric edge removal  
19 optimization. Yet combining the reconfiguration algorithm with the shrink-back  
20 and/or the pair-wise edge removal optimizations requires additional beacon  
21 transmission power considerations.

22       To illustrate an additional power consideration, consider that if the shrink-  
23 back operation is performed, using the power to reach all the neighbors in  $G_s$  does  
24 not suffice. Suppose that the network is temporarily partitioned into two sub-  
25 networks  $G_1$  and  $G_2$ ; for every pair of nodes  $u_1 \in G_1$  and  $u_2 \in G_2$ , the distance

1  $d(u_1, u_2) > R$ . Suppose that  $u_1$  is a boundary node in  $G_1$  and  $G_2$  is a boundary node  
2 in  $G_2$ , and that, as a result of the shrink-back operation, both  $u_1$  and  $u_2$  use power  
3  $P' < P$ . Further suppose that later nodes  $u_1$  and  $u_2$  move closer together so that  
4  $d(u_1, u_2) < R$ . If  $P'$  is not sufficient power for  $u_1$  to communicate with  $u_2$ , then they  
5 will never be aware of each other's presence, since their respective beacons will  
6 not reach each other—thus they will not detect that the network has become  
7 reconnected. In this illustration network connectivity is *not* preserved. This  
8 problem is solved by having the boundary nodes broadcast with the power  $P$   
9 computed by the basic CBTC- $\alpha$  algorithm.

10 In yet another example of a power consideration, consider that with the  
11 pairwise edge removal optimization, it is necessary for  $u$ 's beacon to broadcast  
12 with the power needed to reach all of  $u$ 's neighbors in the basic CBTC- $\alpha$ , not just  
13 the power needed to reach all of  $u$ 's neighbors after the optimization is performed.  
14 This choice of beacon power guarantees that the reconfiguration algorithm works  
15 in combination with the optimization.

16 The reconfiguration protocol works in an asynchronous setting. In  
17 particular, the synchronous model with reliable channels (assumed until now) is  
18 relaxed to allow asynchrony and both communication and node failures. Now  
19 nodes are assumed to communicate asynchronously, messages may get lost or  
20 duplicated, and nodes may fail (although we consider only *crash* failures: either a  
21 node crashes and stops sending messages, or it follows its algorithm correctly).  
22 At any given time, each node  $u$  can examine the messages sent to it, compute, and  
23 send messages using, for example, either the *bcast* or *send* primitives.

24 A limited loss model is assumed where a message will be received  
25 infinitely many times if it is sent infinitely many times, message delay is bounded

for transmissions that are not lost, and messages have unique identifiers and mechanisms to discard duplicate messages are present. The faulty behavior of nodes can only be stopping failure. Messages have unique identifiers and that mechanisms to discard duplicate messages are present. Node failures result in *leave* events, as do lost messages. If node  $u$  gets a message after many messages having been lost, there will be a *join* event corresponding to the earlier *leave* event. The asynchronous exchange algorithm works correctly in this relaxed model.

To illustrate this, consider that even if a node  $v$  fails to receive a beacon request message from  $u$ , it will eventually receive  $u$ 's the periodic NDP beacon message. Based on the transmission power  $pm$  and reception power  $pm'$  of the beacon, the power  $p(d(u, v))$  can be derived. Firstly, consider the basic CBTC algorithm. If there is no failure or mobility after a specific time  $t$ , each node  $u$  will construct the same neighbor set  $N(u)$  as the one in the synchronous model. Therefore, the algorithm will eventually construct the same connected embedded graph  $Gr$  as the one in the synchronous model.

For an algorithm with special edge removal, even if the revocation message from  $u$  to an o-neighbor  $v$  is lost, node  $v$  will eventually remove  $se(v, u)$  since  $v$  will not receive  $u$ 's periodic beacon message  $m$  sent by  $bcast(p(rad(u)), m)u$  and will eventually consider  $u$  failed.

### **Energy Efficiency of Control Traffic**

Since topology control protocol itself consumes power, it is substantially beneficial to keep the power consumption of topology control protocol small. The energy efficiency for the basic cone-based algorithm depends on the

algorithm implemented in *Increase(p)* as shown above in Table 1. The simplest solution is to beacon with the maximum power  $P$  once. However, this could take excessive power.

Assume that  $p(u)$  is the optimal terminating power for the basic cone-based algorithm, if the beacon power is doubled every time, then the terminating power  $p(rad(u))$  is bounded by  $2p(u)$ . Thus this simple scheme achieves a 2-competitive solution in terms of the optimal *Increase(p)* function (the optimal solution is to beacon once using  $p(u)$  assuming a reliable channel).

### Experimental Results

Figs. 10-14 show topology graphs illustrating how the cone-based algorithm and the various optimizations improve network topology. Fig. 10 shows a topology graph in which no topology control is employed and every node transmits with the maximum power. Fig. 11 shows the corresponding graph produced by the cone-based algorithm with  $\alpha = 2\pi/3$ . We can see that the latter allows nodes in the dense areas to automatically reduce their transmission radius. Fig. 12 illustrates the graph after the shrink-back operation is performed, in which some boundary nodes are allowed to reduce their radius. Figs. 13 and 14 are the topology graphs after the special edge removal and pair-wise edge removal are applied, respectively.

Table 2 compares the cone-based algorithm with  $\alpha = 2\pi/3$  and  $\alpha = 5\pi/6$  in terms of average node degree and average radius.



TABLE 2

Average degree and radius of different topology control algorithms

	Basic		with $op_1$		$op_1$ and $op_2$	$op_1, op_2, op_3$	Max Power
Average Node Degree	$\alpha=5\pi/6$	$\alpha=2\pi/3$	$\alpha=5\pi/5$	$\alpha=2\pi/3$	$\alpha=2\pi/3$	$\alpha=2\pi/3$	15.65
	9.58	11.61	8.66	10.59	6.51	2.45	
Average Radius	161	182	153	173	166	108	250

In Table 2  $op_1$  represents the shrink-back algorithm,  $op_2$  represents the special edge removal algorithm, and  $op_3$  represents the pair-wise edge removal algorithm. As expected, the table shows that a larger  $\alpha$  results in smaller node degree and radius. However, it also shows that  $\alpha = 2\pi/3$  allows two more optimizations to be applied and these optimizations are very effective in reducing node degree and radius.

The “Max Power” column of Table 2 provides the performance numbers of the case of no topology control where the transmission power of each node is 250, the maximum. A comparison of the last two columns shows that cone-based algorithm with  $\alpha = 2\pi/3$  and all three optimizations applied achieves an average node degree of 2.45, which is less than one sixth of 15.65, and an average radius of 108, which is less than half of the maximum power.

### Exemplary Wireless Multi-Hop Network Node

Fig. 15 shows an exemplary wireless node 1500 to implement topology control in a wireless multi-hop network. The exemplary wireless node is only one example of a suitable computing environment and is not intended to suggest any limitation as to the scope of use or functionality of an exemplary system and

1 procedure providing topology control to wireless multi-hop networks. The  
2 exemplary wireless node should not be interpreted as having any dependency or  
3 requirement relating to any one or combination of components illustrated in  
4 Fig. 15.

5 The exemplary system and procedure providing topology control to  
6 wireless multi-hop networks is operational with numerous other general purpose  
7 or special purpose computing system environments or configurations. Examples  
8 of well known computing systems, environments, and/or configurations that may  
9 be suitable for use with an system and procedure to provide topology control  
10 include, but are not limited to, personal computers, server computers, thin clients,  
11 thick clients, hand-held or laptop devices, multiprocessor systems,  
12 microprocessor-based systems, set top boxes, programmable consumer electronics,  
13 wireless phones, application specific integrated circuits (ASICs), network PCs,  
14 minicomputers, mainframe computers, distributed computing environments that  
15 include any of the above systems or devices, and the like.

16 The node 1500 includes a processor 1502 that is coupled to a system  
17 memory 1504, a power supply 1506, and a communication unit 1508. The system  
18 memory includes any combination of volatile and non-volatile computer-readable  
19 media for reading and writing. Volatile computer-readable media includes, for  
20 example, random access memory (RAM). Non-volatile computer-readable media  
21 includes, for example, read only memory (ROM), magnetic media such as a hard-  
22 disk, an optical disk drive, a floppy diskette, a flash memory card, a CD-ROM,  
23 and/or the like.

24 The processor is configured to fetch and execute computer program  
25 instructions from application programs 1510 such as Cone-Based Topology

Control (CBTC- $\alpha$ ) module 1512, an operating system (not shown), and so on. The processor also stores and fetches data 1514 such as topology control metrics information 1516 while executing the application programs.

The power supply 1506 provides operational power such as battery power to the node 1500. The communication unit 1508 such as a radio communication unit to send and receive messages to/from other nodes and computing devices.

The CBTC- $\alpha$  module 1512 may considerably increase network lifetime and maintain global connectivity with reasonable throughput in a wireless multi-hop network. Network lifetime is affected by determining efficient transmitting radii for each node in the network to communicate with particular ones of the other network nodes. As discussed in great detail above, this is accomplished while substantially guaranteeing a same maximum connected node set as when all nodes are transmitting with full power. Additionally, in contrast to previous approaches for topology control in multi-hop networks that require node positional information, the CBTC- $\alpha$  module 1512 provides a solution that uses locally obtained directional information of other nodes to provide topology control to the network.

The CBTC- $\alpha$  module 1512 provides topology control to a network in a number of phases, which are summarized as follows. Starting with a small transmission radius, each node (denoted by node  $u$ ) broadcasts a neighbor-discovery message. Each receiving node acknowledges this broadcast message. Node  $u$  records all acknowledgments and the directions they came from. Node  $u$  then determines whether there is at least one neighbor in every cone of  $\alpha$  degrees, centered on Node  $u$ . In this first phase, Node  $u$  continues the neighbor discovering process by increasing its transmission radius (operational power) until either the

1 above condition is met or an optimal termination power  $P$  (e.g., a power that is less  
2 than or equal to the nodes maximum transmission power) is reached. For  $\alpha$   
3 smaller than or equal to  $5\pi/6$ , the algorithm substantially guarantees a maximum  
4 connected node set and power efficient connectivity between various node sets.

5 In an optional second phase, and without impacting node connectivity, the  
6 CBTC- $\alpha$  module 1512 removes special and redundant edges to reduce the node  
7 degrees and thereby reduce signal interference and data throughput.

### 8 **Computer-Executable Instructions**

9 An exemplary system and procedure providing topology control may be  
10 described in the general context of computer-executable instructions, such as  
11 program modules, being executed by a computer. Generally, program modules  
12 include routines, programs, objects, components, data structures, etc. that perform  
13 particular tasks or implement particular abstract data types. An exemplary system  
14 and procedure providing topology control may also be practiced in distributed  
15 computing environments where tasks are performed by remote processing devices  
16 that are linked through a communications network. In a distributed computing  
17 environment, program modules may be located in both local and remote computer  
18 storage media including memory storage devices.

### 19 **Computer Readable Media**

20 An exemplary system and procedure providing topology control may be  
21 stored on or transmitted across some form of computer-readable media.  
22 Computer-readable media can be any available media that can be accessed by a  
23 computer. By way of example, and not limitation, computer readable media may  
24 comprise "computer storage media" and "communications media."  
25

1       “Computer storage media” include volatile and non-volatile, removable and  
2 non-removable media implemented in any method or technology for storage of  
3 information such as computer readable instructions, data structures, program  
4 modules, or other data. Computer storage media includes, but is not limited to,  
5 RAM, ROM, EEPROM, flash memory or other memory technology, CD-ROM,  
6 digital versatile disks (DVD) or other optical storage, magnetic cassettes, magnetic  
7 tape, magnetic disk storage or other magnetic storage devices, or any other  
8 medium which can be used to store the desired information and which can be  
9 accessed by a computer.

10       “Communication media” typically embodies computer readable  
11 instructions, data structures, program modules, or other data in a modulated data  
12 signal, such as carrier wave or other transport mechanism. Communication media  
13 also includes any information delivery media.

14       The term “modulated data signal” means a signal that has one or more of its  
15 characteristics set or changed in such a manner as to encode information in the  
16 signal. By way of example, and not limitation, communication media includes  
17 wired media such as a wired network or direct-wired connection, and wireless  
18 media such as acoustic, RF, infrared, and other wireless media. Combinations of  
19 any of the above are also included within the scope of computer readable media.

## 20   **Exemplary Procedure**

21       Fig. 16 shows an exemplary procedure to provide topology control to a  
22 wireless multi-hop network. At block 1602, each node (e.g., a node 1500 of  
23 Fig. 12) performs node discovery using incoming signals from other nodes in the  
24 network. The Incoming signals are in response to a discover neighbor message  
25

1 communicated by a respective node. At block 1604, each node makes a respective  
2 decision about a substantially optimal transmission power with which to  
3 communicate the one or more of the discovered nodes (block 1602). This decision  
4 is based on the incoming signals. Significantly, this decision is also made  
5 independent of positional information such as latitude and longitude.

6 At block 1606, each node maintains communications with at least one  
7 subset of the discovered nodes to provide connectivity to the distributed wireless  
8 multi-hop network.

### 9 **Conclusion**

10 Although the system and procedure providing topology control to a  
11 wireless multi-hop network has been described in language specific to structural  
12 features and/or methodological operations, it is to be understood that the system  
13 and procedure to system and procedure providing topology control to a wireless  
14 multi-hop network defined in the appended claims is not necessarily limited to the  
15 specific features or operations described. Rather, the specific features and  
16 operations are disclosed as preferred forms of implementing the claimed present  
17 subject matter.